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HYPERSONIC FLOW AROUND A CIRCULAR CONE AT AN ANGLE OF ATTACK

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HYPERSONIC FLOW AROUND A CIRCULAR CONE AT AN ANGLE OF ATTACK *

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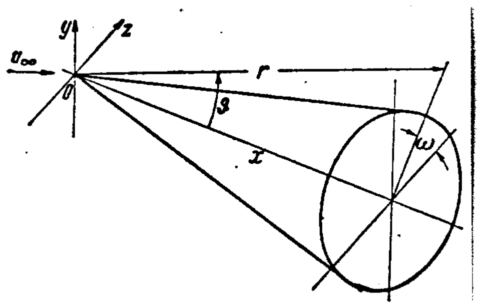
The problem of a cone flown around by a hypersonic gas was solved in the Cheng work [1] by the method of expansion by small parameters $\varepsilon = (\gamma - 1)/(\gamma + 1)$ and $\sigma = \sin\alpha / \sin\tau$, where γ is the heat capacity ratio, α is the angle of attack, τ is the semi-aperture angle of the cone. In the present work the same problem is solved by the same method. A first approximation of the solution is obtained with its correction in the vicinity of the surface of the cone, and a second approximation — for the pressure. But the results obtained show a departure from those of ref. [1].



1. - Let us consider the flow around a circular cone with **semi-aperture** τ by a homogenous hypersonic gas flow at an angle of attack α, in a spherical system of coordinates r, ϑ, ω with the axis coinciding with that of the cone (see Figure).

We shall designate by u_+, v_+, w_+ the velocity vector components in the direction of increase r, ϑ, ω; by p_+, ρ_+ respectively the pressure and density.

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* Giperzvukovoye obtekaniye kryglogo konusa pod uglom ataki.

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We introduce the dimensionless variables as follows :

$$u \equiv \frac{u_+}{u_\infty}, \qquad v \equiv \frac{v_+}{\varepsilon u_\infty \sin\tau}, \qquad w \equiv \frac{w_+}{u_\infty \sin\alpha}$$
$$p \equiv \frac{p_+}{\rho_\infty u_\infty^2 \sin^2\tau}, \qquad \rho \equiv \frac{\varepsilon\rho_+}{\rho_\infty}, \qquad \theta \equiv \frac{\sin\vartheta - \sin\tau}{\varepsilon \sin\tau} \tag{1.1}$$

The quantity of motion, continuity and energy equations will take the form

$$\begin{aligned}
&\left[Iv\frac{\partial}{\partial\theta} + \sigma\frac{w}{1+\varepsilon\theta}\frac{\partial}{\partial\omega}\right]u = \sin^2\tau\,[\varepsilon^2 v^2 + \sigma^2 w^2]\\
&\frac{\partial p}{\partial\theta} = \sigma^2\rho\frac{w^2}{1+\varepsilon\theta} - \varepsilon\rho\left[v\frac{\partial}{\partial\theta} + \sigma\frac{w}{I(1+\varepsilon\theta)}\frac{\partial}{\partial\omega} + \frac{u}{I}\right]v\\
&\sigma\rho\left[IV\frac{\partial}{\partial\theta} + \sigma\frac{w}{1+\varepsilon\theta}\frac{\partial}{\partial\omega} + u\right]w = -\frac{\varepsilon}{1+\varepsilon\theta}\left[\frac{\partial p}{\partial\omega} + \sigma I\rho v w\right]\\
&2(1+\varepsilon\theta)\rho u + I\frac{\partial}{\partial\theta}[(1+\varepsilon\theta)\rho v] + \sigma\frac{\partial}{\partial\omega}(\rho w) = 0\\
&\left[Iv\frac{\partial}{\partial\theta} + \sigma\frac{w}{1+\varepsilon\theta}\frac{\partial}{\partial\omega}\right]\left(\frac{p}{\rho^\gamma}\right) = 0
\end{aligned} \tag{1.2}$$

Here

$$I = \cos\vartheta = [1 - \sin^2\tau\,(1+\varepsilon\theta)^2]^{1/2}$$

The flow around condition at the surface of the cone is

$$v = 0 \quad \text{at} \quad \theta = 0.$$

At ths shock wave surface $\theta = \theta^+(\omega)$ we have the conditions for the preservation

of the mass :

$$I\,[\rho v - I\sigma\sin\omega + (1+\varepsilon\theta^+)\cos\alpha] = \sigma\frac{\theta_\omega^+}{1+\varepsilon\theta^+}[\rho w - \varepsilon\cos\omega]$$

of the impulse

$$\begin{aligned}
\left[I^2 + \left(\frac{\varepsilon\theta_\omega^+}{1+\varepsilon\theta^+}\right)^2\right](p - k\varepsilon) = &\left[I(1+\varepsilon\theta^+)\cos\alpha - I^2\sigma\sin\omega + \sigma\varepsilon\frac{\theta_\omega^+}{1+\varepsilon\theta^+}\cos\omega\right]^2 -\\
&- \varepsilon\rho\left[Iv - \sigma\frac{\theta_w^+}{1+\varepsilon\theta^+}w\right]^2
\end{aligned} \tag{1.3}$$

of energy

$$\begin{aligned}
&\left[I^2 + \left(\frac{\varepsilon\theta_\omega^+}{1+\varepsilon\theta^+}\right)^2\right]\left(\frac{p}{\rho} - k\right)(1+\varepsilon) + \varepsilon^2\left[Iv - \sigma\frac{\theta_\omega^+}{1+\varepsilon\theta^+}w\right]^2 =\\
&= \left[I(1+\varepsilon\theta^+)\cos\alpha - I^2\sigma\sin\omega + \sigma\varepsilon\frac{\theta_\omega^+}{1+\varepsilon\theta^+}\cos\omega\right]^2
\end{aligned}$$

of tangent components

$$\begin{aligned}
&I\sigma(w - \cos\omega) + \varepsilon\frac{\theta_\omega^+}{1+\varepsilon\theta^+}[\varepsilon V - I\sigma\sin\omega + (1+\varepsilon\theta^+)\cos\alpha] = 0\\
&u - I\cos\alpha = \sin^2\tau\,(1+\varepsilon\theta^+)\,\sigma\sin\omega
\end{aligned}$$

Here

$$k = \frac{\gamma + 1}{\gamma(\gamma - 1) M_\infty^2 \sin^2 \tau}, \qquad \theta_\omega{}^+ = \frac{d\theta^+(\omega)}{d\omega}$$

2. — We seek the solution in the form of series by ϵ and σ of the type

$$p = p_{00} + p_{10}\varepsilon + p_{01}\sigma + p_{20}\varepsilon^2 + p_{11}\varepsilon\sigma + p_{02}\sigma^2 + \cdots \tag{2.1}$$

The function $\theta^+(\omega)$, characterizing the shock wave position, is also sought in the form

$$\theta^+ = \theta_{00} + \theta_{10}\varepsilon + \theta_{01}\sigma + \theta_{20}\varepsilon^2 + \theta_{11}\varepsilon\sigma + \theta_{02}\sigma^2 + \cdots \tag{2.2}$$

The quantity k is considered limited.

Substituting the type (2.1), (2.2) expansions into the equations and boundary conditions, we shall assemble the terms with identical powers ε and σ, and obtain the equations for series' (2.1) and (2.2) coefficients. The solutions of the obtained equations are materialized in elementary functions. Let us bring forth the solution in the first approximation

$$\begin{gathered}
u = \cos\tau - \varepsilon\left(\frac{1+k}{2}\right)\sin\tau\,\mathrm{tg}\,\tau + \sigma\sin^2\tau\sin\omega \\
v = -2\theta + \varepsilon\left[\theta(1+k)\,\mathrm{tg}^2\tau + \theta^2(1-\mathrm{tg}^2\tau) - \frac{4\theta^3}{3(1+k)}\right] + \\
+ \sigma\frac{\sin\omega}{\cos\tau}\left[\frac{1+k}{3}\left(\frac{2\theta}{1+k}\right)^{3/2} - 2\theta\sin^2\tau\right] \\
w = \cos\omega\left(\frac{2\theta}{1+k}\right)^{1/2} + \varepsilon\cos\omega\left\{2(1+k) + \left(\frac{2\theta}{1+k}\right)^{1/2}\left[\frac{k^2}{2(1+k)} - k + \right.\right. \\
\left. + \frac{1+k}{3\cos^2\tau} - \frac{15}{8}(1+k) - \frac{1+k}{4}\,\mathrm{tg}^2\tau\right] + \left(\frac{2\theta}{1+k}\right)^{3/2}\left[\frac{1+k}{8}\,\mathrm{tg}^2\tau - \frac{3}{8}(1+k)\right] - \\
\left. - \left(\frac{2\theta}{1+k}\right)^{5/2}\frac{1+k}{24}\right\} + \sigma\frac{\sin 2\omega}{\cos\tau}\left[\frac{1}{2}\left(1 - \frac{k}{1+k}\cos^2\tau\right)\left(\frac{2\theta}{1+k}\right)^{1/2} - \frac{1}{3}\left(\frac{2\theta}{1+k}\right)\right] \\
p = 1 + \varepsilon\left(\frac{1+5k}{4} - \frac{\theta^2}{1+k}\right) - \sigma\, 2\cos\tau\sin\omega \\
\rho = \frac{1}{1+k} + \varepsilon\left[\frac{1}{4} + \left(\frac{k}{1+k}\right)^2 - \left(\frac{\theta}{1+k}\right)^2\right] - \sigma\frac{2k}{(1+k)^2}\cos\tau\sin\omega \\
\theta^+ = \frac{1+k}{2} + \varepsilon\left[\frac{(1+k)^2}{24}(7 + 3\,\mathrm{tg}^2\tau) - \frac{k^2}{2}\right] + \sigma\frac{\sin\omega}{\cos\tau}\left[k\cos^2\tau - \frac{1+k}{3}\right]
\end{gathered} \tag{2.3}$$

The obtention of higher order approximations offers no difficulties and are obtained analogously. In particular, the second approximation for the pressure has the form

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$$
\begin{aligned}
p = {} & 1 + \varepsilon\left(\frac{1+5k}{4} - \frac{\theta^2}{1+k}\right) - \sigma\, 2\cos\tau\sin\omega + \varepsilon^2\left[\frac{3}{32}(1+k)^2 - \frac{k^2}{4} + \right.\\
& + \frac{\mathrm{tg}^2\,\tau}{4}(1+k)^2 + \frac{(2\theta)^2}{4}\mathrm{tg}^2\,\tau - \left(\frac{\theta k}{1+k}\right)^2 - \frac{(2\theta)^2}{16} + \frac{5\,(2\theta)^3}{24\,(1+k)} - \\
& \left. - \frac{(2\theta)^3\,\mathrm{tg}^2\,\tau}{8\,(1+k)} - \frac{11}{96}\frac{(2\theta)^4}{(1+k)^2}\right] + \sigma^2\left[\cos 2\tau - \cos^2\omega \times \right. \\
& \left. \times \left(\cos^2\tau + \frac{1}{4} - \frac{\theta^2}{(1+k)^2}\right)\right] + \sigma\varepsilon\frac{\sin\omega}{\cos\tau}\left\{\frac{4}{15}(1+k)\left[\left(\frac{2\theta}{1+k}\right)^{1/2} - 1\right] + \right.\\
& \left. + \frac{\sin^2\tau}{2}\left[1 - \frac{(2\theta)^2}{1+k}\right] + \frac{k}{2}\left[\left(\frac{2\theta}{1+k}\right)^2\cos^2\tau + 1\right]\right\}
\end{aligned}
\tag{2.4}
$$

From (2.4) we shall obtain the coefficients for the normal and longitudinal forces

$$
\begin{aligned}
C_N &= \frac{2N}{\rho_\infty u_\infty^2 \pi r^2 \sin^2\tau} = \sin\alpha\left\{2\cos^2\tau + \varepsilon\left[\frac{4}{15}(1+k) - \frac{\sin^2\tau}{2} - \frac{k}{2}\right]\right\}\\
C_X &= \frac{2X}{\rho_\infty u_\infty^2 \pi r^2 \sin^2\tau} = 2\sin^2\tau\left\{1 + \varepsilon\left(\frac{1+5k}{4}\right) + \right.\\
&\quad \left. + \sigma^2\left(\frac{3\cos^2\tau}{2} - \frac{9}{8}\right) + \varepsilon^2\left[\frac{3}{32}(1+k)^2 - \frac{k^2}{4} + \frac{\mathrm{tg}^2\,\tau}{4}(1+k)^2\right]\right\}
\end{aligned}
\tag{2.5}
$$

3. — The entropy, computed by the results in the first approximation, takes a variable value at the surface of the cone, which implies, that the solution obtained by the given method is not valid in the vicinity of the surface of the cone [2, 3].

It is shown in [1] that the entropy may be represented in the first approximation in the form

$$S = S_0 + \varepsilon S_{10} + \sigma S_{01}(\zeta) + O(\sigma^2)$$

Here

$$\zeta = \theta^{\sigma\varepsilon\,(1+k)\sec\tau}\,\mathrm{tg}\left(\frac{1}{2}\omega + \frac{1}{4}\pi\right) \tag{3.1}$$

From the boundary condition for entropy we can determine the constants S_0, S_{10} and the function $S_{01}(\zeta)$. The computations give

$$\frac{p}{\rho^\gamma} = 1 + k + \varepsilon\,[k + 2(1+k)\ln(1+k)] + 2\sigma\cos\tau\,\frac{1-\zeta^2}{1+\zeta^2} \tag{3.2}$$

As has been shown in [3], the expressions for w_0 and v_0 are also valid in the vicinity of the cone's surface. The expression (2.4) for the pressure **p** is correct with a precision to magnitudes of the second order of smallness. We then have from (3.2) and (2.3) :

$$\rho = \frac{1}{1+k} + \varepsilon\left[\frac{1}{4} + \left(\frac{k}{1+k}\right)^2 - \left(\frac{\theta}{1+k}\right)^2\right] - \frac{2\sigma\cos\tau}{(1+k)^2}\left[\frac{1-\zeta^2}{1+\zeta^2} + (1+k)\sin\omega\right] \tag{3.3}$$

Utilizing the Bernoulli integral, we obtain

$$u = \cos\tau - \frac{\varepsilon}{2}(1+k)\sin\tau \operatorname{tg}\tau - \sigma \sin^2\tau \frac{1-\zeta^2}{1+\zeta^2} \qquad (3.4)$$

Beyond the vicinity of the cone's surface the expressions (3.2) and (3.4) pass into the expressions from (2.3) corresponding to them. We obtained in this way the first approximation for the solution of the problem of hypersonic gas flow around a circular cone. In the expressions obtained here for the density, shape of the shock wave, circular celocity and pressure, the terms with n (1 + k) are absent. This is where the departure mainly manifests itself from the results of the work [1], and at the same time, while this term is present in the expression for entropy, it is absent in the Cheng's work.

In the particular case when $\sigma = 0$, the obtained first approximation corresponds to the results of the work [4] by Chernyy. It is probable that the error in Cheng's results occurred because of incorrectly computed entropy.

In conclusion, I am grateful to B. M. Bulakh for setting up the problem and for the discussion of the results.

***** THE END *****

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REFERENCES

1. CHENG H. K. Hypersonic Flows past a Yawd Circular Cone and other Pointed Bodies. - Journ of Fluid Mechanics, Vol. 12, p.2, Feb.1962.

2. FERRI A. - Supersonic Flow around Circular Cones at Angles of Attack.- NACA Report No. 01045, 1951.

3. BULAKH B. M. Sverkhzvukovoy potok okolo naklonennogo krugovogo konusa (Spersonic Flow near an Inclined Circular Cone.- Prikl.Matem.i Mekh., (PMM), T.26, v.2, 1962

4. CHERNYY G. G. Techeniye gaza s bol'shoy sverkhzvukovoy skorost'yu. (High-velocity Supersonic Gas Flow). Fizmatgiz, 1959.